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A NOTE ON FREE CONVECTION IN TURBINE CAVITIES

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A study is made of free convection in axisymmetric cavities with variable acceleration forces. It is shown that convective heat transfer should be taken into account when dealing with turbine rotor cavities.

Free convection is a matter of importance in various branches of engineering. The thermal convection process is described by the equations [1]:

$$\dot{\bar{v}} + (\bar{v}\nabla)\bar{v} = -\frac{\nabla p}{\rho} + \bar{g}\beta\vartheta + \nu\Delta\bar{v}, \tag{1}$$

$$\dot{\vartheta} + \bar{v}\cdot\nabla\vartheta = \varkappa\Delta\vartheta, \tag{2}$$

$$\dot{\rho} + \nabla(\rho\bar{v}) = 0. \tag{3}$$

There are no known methods of obtaining exact solutions to nonlinear equations such as (1)–(3). For this reason, the mathematical side of the problem has seen little development, and efforts have been devoted almost exclusively to existence proofs and the elucidation of conditions for stable motion in various special cases [2, 3].

The known experimental investigations for free convection in an internal cavity pertain mainly to constant-temperature walls, and are limited to an evaluation of total heat flux and a qualitative description of convective processes; no quantitative description is attempted [4, 5].

Ostroumov [6] has made a detailed theoretical and experimental study of thermal convection in cylindrical vertical and inclined cavities by linearizing the governing equations. Interesting results have also been obtained by other researchers, but in many cases the data accumulated are insufficient.

There is thus little possibility of an accurate quantitative calculation for free convection phenomena in the cavities of heat engines, and primarily in gas and steam turbines, where their role is all the more important due to the temperature increase of the working medium. It is therefore not possible to make a proper evaluation of temperature distributions in units and components operating in the hottest zone, where temperature nonuniformities may lead to very considerable supplementary stresses, the permissible value of which is also determined by the temperature level. Moreover, in certain cases an underestimate of the effectiveness of convective heat removal may lead to structural complications due to the introduction of special artificial cooling.

The formulation of the problem of free convection in turbines cavities is extremely complicated, mainly because of the diversity of cavity shapes and temperature boundary conditions. This leads to very low efficiency of the experimental methods of investigation, since the results obtained may be applied only to a narrow range of problems. In fact, both the geometrical size of the cavity and the temperature conditions at its boundaries must be similar to those involved in the test.

By taking into account a number of special features peculiar to the given case, certain simplifications may be made in formulating the problem. Parametric linearization of the initial conditions is permissible. Since in most cases free convection is strongest under steady operating conditions, we may begin by considering the steady problem. The nearly axisymmetric nature of the processes in turbines often makes it possible to heat the process as two-dimensional rather than three-dimensional.

In turbine rotor cavities the process is clearly axisymmetric. However, the solution of the free convection problem is complicated by the fact that in this case centrifugal and not gravitational forces act as the body forces causing convection, and these depend on the distance $\bar{r}$ to the axis of rotation. Thus, $\bar{g} = \omega^2\bar{r}$ in (1).

We may now introduce the stream function ψ, so that (3) is satisfied, and taking the curl of both sides of (1), eliminate the pressure.

Choosing some geometric length R and some temperature difference Θ as characteristic values, we may put the equations in the dimensionless form:

$$\frac{1}{r}\left(\frac{\partial^4\psi}{\partial r^4}+2\frac{\partial^4\psi}{\partial r^2\partial z^2}+\frac{\partial^4\psi}{\partial z^4}\right)+\frac{1}{r^2}\left[\frac{\partial\psi}{\partial r}\left(\frac{\partial^3\psi}{\partial r^2\partial z}+\frac{\partial^3\psi}{\partial z^3}\right)-\right.$$

$$\left.-\left(2+\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial^3\psi}{\partial r^3}+\frac{\partial^3\psi}{\partial r\partial z^2}\right)\right]+\frac{1}{r^3}\left[3\frac{\partial^2\psi}{\partial r^2}\left(1+\frac{\partial\psi}{\partial z}\right)-\right.$$

$$\left.-\frac{\partial^2\psi}{\partial r\partial z}\frac{\partial\psi}{\partial r}+2\frac{\partial^2\psi}{\partial z^2}\frac{\partial\psi}{\partial z}\right]-\frac{3}{r^4}\frac{\partial\psi}{\partial r}\left(1+\frac{\partial\psi}{\partial z}\right)+$$

$$+r\,\mathrm{Gr}\frac{\partial t}{\partial z}+2\left(\mathrm{Re}_\omega+\frac{\mathrm{Re}_u}{r}\right)\frac{\partial\,\mathrm{Re}_u}{\partial z}=0, \tag{4}$$

$$\frac{\partial^2 t}{\partial r^2}+\frac{\partial^2 t}{\partial z^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{\mathrm{Pr}}{r}\left(\frac{\partial\psi}{\partial r}\frac{\partial t}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial t}{\partial r}\right)=0, \tag{5}$$

$$\frac{\partial^2\mathrm{Re}_u}{\partial r^2}+\frac{\partial^2\mathrm{Re}_u}{\partial z^2}+\frac{1}{r}\left(\frac{\partial\psi}{\partial r}\frac{\partial\,\mathrm{Re}_u}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\,\mathrm{Re}_u}{\partial r}\right)-$$

$$-\frac{\mathrm{Re}_u}{r^2}\left(1+\frac{\partial\psi}{\partial z}\right)+\frac{1}{r}\left(\frac{\partial\,\mathrm{Re}_u}{\partial r}-\mathrm{Re}_\omega\frac{\partial\psi}{\partial z}\right)=0. \tag{6}$$

Here Re_ω is the Reynolds number based on the circular rotor velocity at radius R, and Re_u corresponds to the circular component of local velocity of the medium relative to the rotor, which exists even when the process is completely axisymmetric. This component appears only because of convective radial displacements of the medium, and in most cases will be small compared to the other velocity components of the medium and to the transport velocity.

There is reason to suppose that Re_u has practically no influence on the value of the normal temperature derivatives at the boundaries of the region, which are of greatest interest since it is they that determine the heat flux. For these cases the system becomes even simpler, since we may then neglect the last term in (4) and confine the examination to (4) and (5).

If this system is solved, it is easy to determine Re_u from (6), and afterwards to make the solution more exact, if need be, by taking into account the last term of (4).

Substitution of $\mathrm{Gr}t = \bar{t}$ allows us to obtain a system containing only Pr as parameter, but its solution also depends on Gr, which determines the scale of the boundary conditions with respect to temperature.

Examination of actual rotors indicates that it will generally be permissible to make the approximation that the cross section is a rectangle with sides parallel to the radial and axial directions.

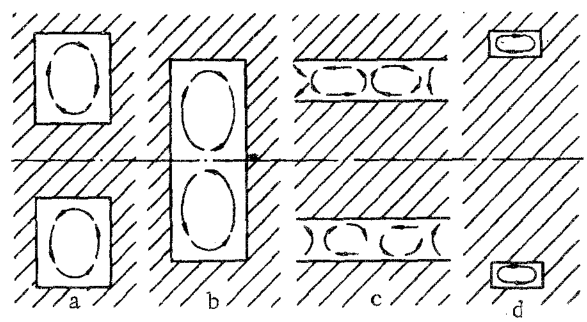

Fig. 1. Some forms of cavities corresponding to different boundary conditions: a) annular cavity with radial and axial dimensions nearly the same; b) axis of rotation passes through cavity; c) one of the cavity dimensions considerably smaller than the other; d) radial dimension of cavity small compared with distance to axis of rotation.

The boundary conditions for the problem depend on the ratio of the radial and axial dimensions of the cavity and on its location relative to the axis of rotation (Fig. 1):

a) The velocities are zero at all boundaries of the region, and so therefore are the first derivatives of the stream function along the boundary and the normal to it. The boundary conditions with respect to temperature are determined by the thermal state of the cavity walls.

b) Because of symmetry we may consider the region corresponding to half the cross section of the cavity; there is then no flow of heat or medium across the axis, and the axial velocity component has an extreme value on the axis. At the other three boundaries of the region conditions are the same as for a).

c) Theoretical and experimental investigations [3, 4] have shown that for plane layers with constant temperature at each wall, the convective motion takes the form of separate closed contours, i.e., cells, over a wide range of conditions. Under very diverse conditions the maximum cell dimension proves to be approximately twice the minimum (layer thickness). For plane horizontal layers the cells are hexagonal in plan, but it may be assumed that under conditions of axial symmetry the convective motion will be cylindrical and that the cell may be regarded as two-dimensional.

Even if this assumption proves to be incorrect, it will clearly be possible to choose ratios of minimum to maximum cell dimensions for which heat flux calculated on a two-dimensional basis will agree with experimental results.

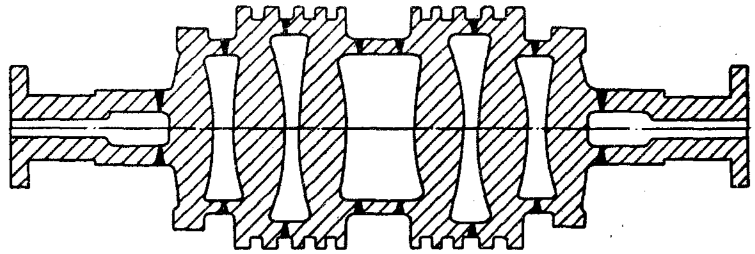

Fig. 2. Turbine rotor.

The boundary conditions at the walls will then be the same as for a), and at the boundaries between cells they are similar to the conditions on the axis of rotation for b). The heat flow from cell to cell, which may occur when the temperature distribution along the walls is nonuniform, will be negligibly small, since there is no convective heat transfer between cells.

d) We may proceed to examine the problem in rectangular coordinates with constant acceleration of the body forces. The same scheme is also valid for convection in the cavities of nonrotating elements.

To derive the solution of (4)–(6) for any form of boundary conditions, we may apply finite-difference methods, which have been widely used in connection with electronic computers. Constraints on the shape of the region may be removed. Finite difference methods minimize the value of the "zero" approximation, which plays a key role in the method of successive approximations [6]. In principle, it is also possible to solve the initial system of equations (1)–(3) on a computer, but this formulation of the problem is still unjustifiably complicated.

In the simplest cases it may also prove expedient to derive a solution in series form.

Transition from the laminar to the turbulent regime brings a radical change in the nature of the convective motion and heat transfer in cavities. However, experimental data [5, 7] show that the dependence of heat transfer on the Gr number is not appreciably affected. Thus, we may expect satisfactory accuracy in evaluating heat transfer not only for laminar, but also for turbulent convection. If need be, we may allow for energy dissipation [1] by introducing an additional term in (2) and (5).

In conclusion, it should be noted that the acceleration in the rotors of modern steady-state turbines reaches values approximately 5000 times that of gravity. On this basis one must expect highly intense heat transfer. A preliminary evaluation of convection in the cavity of the welded rotor of a turbine from the Khar'kov turbine plant (Fig. 2), according to the formulas for a plane layer, at an acceleration corresponding to the mean radius of the cavity and a moderate temperature difference (30°C) between the discs and the cavity walls indicated an increase in effective thermal conductivity by a factor of more than two hundred over that for the medium (air) at rest under the same conditions. This value is only about four times less than the thermal conductivity of the rotor metal in the given temperature interval, and has a very marked influence on the temperature field of the discs.

These data are evidence of the need to allow for thermal convection in closed turbine cavities.

NOTATION

$\bar{v}$–velocity vector of medium; ρ–density of medium; p–pressure; $\bar{g}$–acceleration of body forces; ϑ–temperature, referred to some mean value for the case considered; β, ν, $\varkappa$–coefficient of thermal expansion, kinematic viscosity, and thermal diffusivity; ω–angular velocity of rotation of rotor; r, z–dimensionless coordinates; t–dimensionless temperature; Pr–Prandtl number; Gr–Grashof number, calculated from the acceleration of centrifugal forces at radius R.

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